Surface Wave Techniques to Evaluate Subsurface Stiffness Structure
Fundamentals of Surface Waves
Surface Waves and Body Waves

- Compression Wave or Primary Wave (P Wave)
- Shear Wave or Secondary Wave (S Wave)

Body Waves

Surface Waves

- Love wave
- Rayleigh wave
Propagation of Stress Waves

Body Waves (P or S Waves)

Surface Waves (Rayleigh Waves)
Visualization of Surface-Wave Propagation
Variation of Vertical and Horizontal Displacements with Depth

Normalized Motion = $\frac{\text{Amplitude at Depth } z}{\text{Amplitude at Surface}}$
Propagation Velocities of Stress Waves

Constrained Compression Wave, $V_P$

Shear Wave, $V_S$

Rayleigh Wave, $V_R$
Dispersion Phenomenon of Surface Waves

- Freq = 66.48 Hz
  - Wavelength = 2.0 m
  - $V_{ph} = 133.0 \text{ m/sec}$

- Freq = 15.8 Hz
  - Wavelength = 9.26 m
  - $V_{ph} = 146.4 \text{ m/sec}$

- Freq = 11.9 Hz
  - Wavelength = 17.0 m
  - $V_{ph} = 202.4 \text{ m/sec}$

Half-Space

Phase-Velocity Dispersion Curve

- $V_{ph}$: Phase Velocity
- Wavelength, m
- Phase Velocity, m/sec
Fundamental and Higher Modes of Surface Waves
Apparent Surface-Wave Velocity
Near-Field Effects

Waves
- Generated by a point source on the ground surface.
- Measured by receivers in the vicinity of the source.

Near-field effects
• Body wave interference
• Cylindrical wave front of Rayleigh waves

Body wave interference
- Body wave components and surface wave components not well separated at this stage.
  As the wave train travels away from the source, the relative contribution of body wave components decrease so that in the far-field it is acceptable to neglect the influence of body waves.

Spreading of surface waves
- The wave front is assumed to be plane in determining phase velocities by most techniques.
  This assumption is valid only in the far field, whereas the cylindrical shape of the wave front cannot be neglected and accurate analysis requires the use of transforms employing cylindrical coordinates.

• Low-frequency components exhibit more near-field effects, requiring the adoption of adequate countermeasures.
• Typical strategy is to increase the offset of the first receiver of the array with respect to the source position.
Comparison of Short- and Long-Array Measurements

Wave groups in wave propagation in short and long arrays

**Long Array**

(a) $R_1 = 45\text{m}$

(b) 
- Wave group in soil layer
- Wave group in bedrock layer

**Short Array**

(c) $R_1 = 48\text{m}$

(d) 
- Wave group in soil layer
Multiple Sources

- Biased beamformer:
  - $S=3m$
  - $S=24m$
  - $S=48m$

- Zywicki (1999):
  - $R_1=0.94m$, $R_{15}=60m$

- Fundamental-Mode Vel.
  - Higher-Mode Vel.

- Note:
  - $S=Source$ offset
  - $R_1=Src$ to the 1st rec. distance

- Note:
  - $d=Receiver$ Spacing
  - $S=Source$ offset

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Unbiased beamformer:

- $d=1m$, $S=3m$
- $d=1m$, $S=24m$
- $d=1m$, $S=48m$

- Receivers for unbiased array
  - CapSASW Tests: $R_1$, $R_4$
  - SBF Beamforming Tests: $R_1$, $R_2$, $R_3$, $R_4$

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*Note:

- Source
  - Receivers
Principles of Surface Waves
Analysis Procedure of Surface-Wave Methods

Measurement of Surface Waves [Time or Frequency Domain]

Determination of Phase-Velocity Dispersion Curves

- Inversion Analysis
- Iterative Forward Modeling Analysis

Determination of Shear-Wave Velocity Profile

Analysis Flow of Surface-Wave Measurements

Measurement of Surface-Wave Phase Velocities

Inversion Analysis → Forward Modeling of Surface-Wave Propagation

Determination of 1-D Shear-Wave Velocity ($V_s$) Profile

A Series of 1-D $V_s$ Profiling (10~20 Profiles)

Construction of 2-D Shear-Wave Velocity Contour Using a series of 1-D $V_s$ Profiles

Analysis Procedure of Surface Wave Methods
Surface-Wave Methods

- Spectral-Analysis-of-Surface-Waves (SASW) Method
- Multichannel Analysis of Surface Waves (MASW) Method
- Short-Array Beamforming (SBF) Method
- Continuous Surface Wave (CSW) Method
- Refraction Microtremor (ReMi) Method
- Passive Method
- f-k Spectrum Method
1. SASW Method

(a) Impact, Swept Sinusoidal Vibration, or Random Noise

(b) D=1m

(c) Experimental Dispersion Curve
2. MASW Method

Field Record (offset-time):

\[ u(x,t) \]

FFT along time axis (offset-frequency):

\[ U(x, w) = \int u(x, t) e^{jwt} dt \]

Amplitude Term: \( A(x, w) \)

\[ U(x, w) = P(x, w)A(x, w) \]

Phase Term: \( P(x, w) \)

Amplitude Normalization:

\[ \left[ \frac{A(x, w)}{|A(x, w)|} \right] \]

Phase Shift for Testing Velocities:

\[ \phi = \Phi = \frac{w}{c_w} \]

Loci of Maxima = Dispersion Curve

\[ V(w, \phi) = \int e^{i\phi} \left[ \frac{U(x, w)}{|U(x, w)|} \right] dx \]

\[ = \int e^{-\Phi x} \left[ \frac{A(x, w)}{|A(x, w)|} \right] dx. \]

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**Synthetic Data**

**\( U(x, w) \)**

**\( V(w, \phi) \)**

**\( I(w, c_w) \)**
3. Time-Frequency Plot Method

Time-Frequency Plot for time history h1

Time-Frequency Plot for time history h2

Analytical Function, $A_1$ for frequency = 5 Hz

Analytical Function, $A_2$ for frequency = 5 Hz

Moment's method
Multiple filter/Cross-correlation method
Multiple filter/Time-variable filter method
Frequency-Wave number Method (Joh, 2003)
HWAW Method

Phase-velocity dispersion curve
4. CSW Method

Harmonic-Wave Vibration (frequency = \( f \))

Best-Fit Line

Distance from Source

Phase Velocity:

\[ v_{ph} = 2\pi f \frac{\Delta r}{\Delta \phi} \]
Calculation and Analysis of CWS Method:
Wave number restoration technique
Example of CSW Measurements

![Diagram of CSW Measurements](image)

- **Modes of Rayleigh-Wave Vel.**
- **CSWS Vel.**
- **SASW Vel. (Rec. Pair: R3-R2)**
- **Avg. of SASW Vel.**
- **Avg. of Phase-Unwrapping Vel. (w/o Masking)**

**Vibrator**
**Dynamic Signal Analyzer**
**Four Geophones**
**Amplifier for Vibrator**
**Vibrator**
5. ReMi Method

Domain Transformation
- $p-\tau$ Transform
- Fourier Transform

Velocity Spectral Analysis

Velocity Modeling
### Comparison of Surface-Wave Methods (1)

Key features of four, widely used surface-wave methods (Stokoe, et al., 2004)

<table>
<thead>
<tr>
<th>Surface-Wave Method</th>
<th>Key Features</th>
</tr>
</thead>
</table>
| SASW method         |  - phase velocities from phase differences  
                      - two to four receivers typically used  
                      - superposed-mode phase velocity (apparent phase velocity)  
                      - global property over receiver-spread area  
                      - shear-wave velocity profile from the apparent phase velocities (1-D or 2-D)  
                      - comprehensive forward modeling or inversion analysis  
                      - impulsive source, swept-sine source, or random vibration source |
| f-k spectrum method |  - phase velocities from frequency-wave number spectrum  
                      - multiple receivers (e.g. 128, 256, etc. receivers)  
                      - fundamental and higher-mode phase velocities  
                      - global property over receiver-spread area  
                      - shear-wave velocity profile from fundamental and higher modes (1-D)  
                      - impulsive source |
| MASW method         |  - limited number of receivers (usually 24 receivers)  
                      - fundamental and higher-mode phase velocities  
                      - walk-away measurement  
                      - same measurement configuration as common-midpoint reflection survey  
                      - global property over receiver-spread area  
                      - shear-wave velocity profile from the fundamental mode (1-D or 2-D)  
                      - impulsive source or swept-sine source |
| CSW method          |  - phase velocity from the average phase-angle slope over receiver-spread area  
                      - four to six receivers used  
                      - superposed-mode phase velocity (apparent velocity)  
                      - global property over receiver-spread area  
                      - shear-wave velocity profile from the apparent velocities (1-D)  
                      - steady-state harmonic source |
## Comparison of Surface-Wave Methods (2)

### Advantages and disadvantages of four, widely used surface-wave methods (Stokoe et al, 2004)

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>SASW method</td>
<td>• good sampling of shallow material</td>
<td>• multiple measurements using different source-receiver configurations are required</td>
</tr>
<tr>
<td></td>
<td>• more sensitive measurements for layer stiffness contrast, using apparent velocity inversion analysis</td>
<td>• expertise required for phase unwrapping and forward modeling</td>
</tr>
<tr>
<td>f-k method</td>
<td>• dispersion curves separated for fundamental and higher modes</td>
<td>• aliasing problem in wave number domain</td>
</tr>
<tr>
<td></td>
<td>• body-wave effect extracted</td>
<td>• inaccurate mode separation in case of poor resolution in f-k spectrum</td>
</tr>
<tr>
<td></td>
<td>• dispersion curve global to the receiver-spread area</td>
<td>• large number of traces required for good resolution in wave-number domain</td>
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<tr>
<td></td>
<td></td>
<td>• limitation due to topographic constraint and instrumentation capability</td>
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<tr>
<td></td>
<td></td>
<td>• long measurement time</td>
</tr>
<tr>
<td>MASW method</td>
<td>• mode separation of surface waves</td>
<td>• aliasing problem in wave-number domain</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• use of the fundamental mode only in inversion analysis</td>
</tr>
<tr>
<td>CSW method</td>
<td>• the effects of local anomalies minimized with the use of average phase-angle slope</td>
<td>• dedicated inversion analysis required but not used</td>
</tr>
<tr>
<td></td>
<td>• no expertise required to calculate phase velocity</td>
<td>• near-field effects included</td>
</tr>
<tr>
<td></td>
<td>• reliable measurements with controlled source</td>
<td>• exploration depth limited</td>
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<td></td>
<td></td>
<td>• frequency-content of vibrator is limited</td>
</tr>
</tbody>
</table>
Measurements of Surface Waves
Receivers

Geophones

![Image of geophones and schematic diagram]

![Graph showing response curve of seismic detector]
Accelerometers / MEMS
Data Acquisition System

- Anti-aliasing filter
- 16/18/24-bit resolution
- Triggering
- Dynamic Range
Frequency Aliasing by Undersampling

When the sampling interval is greater or equal to half of the period,

$$\Delta t \geq \frac{T_0}{2}$$

The signal is undersampled and its periodicity appears “aliased” into a signal of lower frequency content.
Frequency of the continuous periodic signal is properly identified from the discrete signal if the sampling frequency \( f_{samp} \) exceeds the Nyquist frequency \( f_{Nyq} \):

\[
f_{samp} = \frac{1}{\Delta t} > f_{nyq} = \frac{2}{T_0}
\]

- In practice, a minimum of about 10 points per cycle is recommended.
- The highest expected frequency should be considered when selecting the sampling rate.
- Analog antialiasing filters must be placed in series before digitization to remove frequency components higher than \( 1/(2\Delta t) \).
Seismic Sources
Cables
Thank you for you attention !!!

감사합니다.